MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

ETM 4096 - DIGITAL SIGNAL PROCESSING

(RE/TE)

29 FEBRUARY 2020
9:00 A.M- 11:00 A.M.
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 11 pages (including cover page) with 4 questions and 6-page list of formulas given in the appendix.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer Booklet provided.

QUESTION 1

(a) A system is governed by the difference equation

$$y[n] = 2x[n] - x[n-3].$$

Determine analytically whether the system:
(i) is linear
(ii) is time-invariant

[3+3 Marks]

Now, consider the impulse response of a different system, which is linear and time-invariant:

$$h[n] = 2^n u[n+1].$$

(iii) Is the system causal? Justify your answer.
(iv) Is the system bounded-input bounded-output (BIBO) stable? Justify your answer.

[2+2 marks]

(b) Consider a causal Linear Time-Invariant (LTI) system that is characterized by the difference equation

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$

(i) Determine the frequency response, $H(e^{j\omega})$ and then the impulse response, $h[n]$.

(ii) If the input to this system is $x[n] = \left(\frac{1}{4}\right)^n u[n]$, determine the system response, $y[n]$.

[7+8 Marks]

Continued.....

QUESTION 2

(a) Determine all possible discrete-time signals $y[n]$ associated with the following z-transform

$$Y(z) = \frac{2z^{-1}}{(1-\frac{1}{2}z^{-1})(5-z^{-1})}$$

[9 marks]

(b) Pole-zero plot of a discrete-time system is shown in **Figure Q2.**

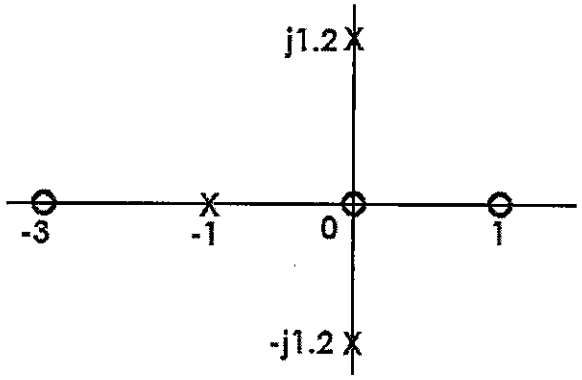


Figure Q2

(i) Determine the system function $H(z)$ of the system. [3 marks]

(ii) Express the difference equation relating the system input $x[n]$ to output $y[n]$.

[5 marks]

(c) Consider the causal, linear, time invariant system described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n]$$

Determine whether the system is stable. Prove your answer using $H(z)$.

[8 marks]

Continued .....

QUESTION 3

(a) Using the Kaiser window function, design a causal highpass FIR filter which satisfies the following requirements:

$$|H(e^{j\omega})| \leq 0.0001, \quad 0 \leq \omega \leq 0.2\pi$$
$$0.999 \leq |H(e^{j\omega})| \leq 1.001, \quad \omega \geq 0.4\pi$$

In specific,

i) Determine the values of ω_c, parameter A, and shape parameter β.

[3 marks]

ii) Find the required order of the filter.

[4 marks]

iii) Write the impulse response of the causal lowpass FIR filter.

[4 marks]

(b) **Figure Q3** shows the desired specification of a digital lowpass filter.

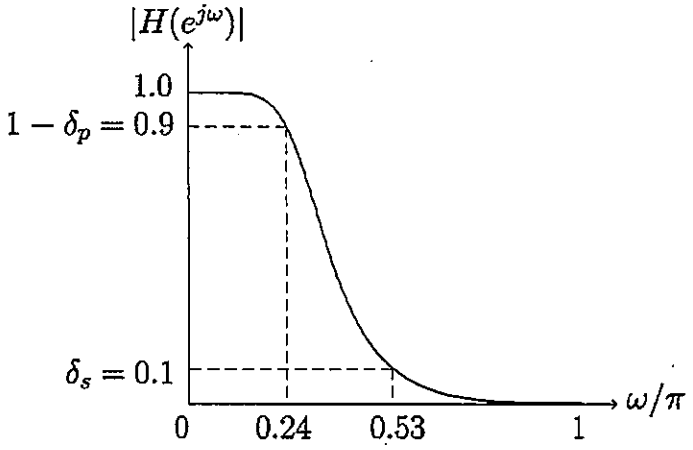


Figure Q3

Using a Butterworth lowpass filter, design a digital IIR filter to satisfy the given specification by using bilinear-transform method. Use $\omega_c = 0.3\pi$,

i. Compute the required order of the filter.

[6 marks]

ii. Determine the system function of the analog prototype filter $H_a(s)$.

[5 marks]

iii. Write the frequency response of the digital IIR filter $H(z)$.

[3 marks]

Continued .....

QUESTION 4

Figure Q4.1 illustrates how Discrete Fourier Transform (DFT) is used to perform linear convolution of two finite sequences $x[n]$ and $y[n]$. Consider $x[n] = \{2,1,-1\}$ and $y[n] = \{-1,1\}$.

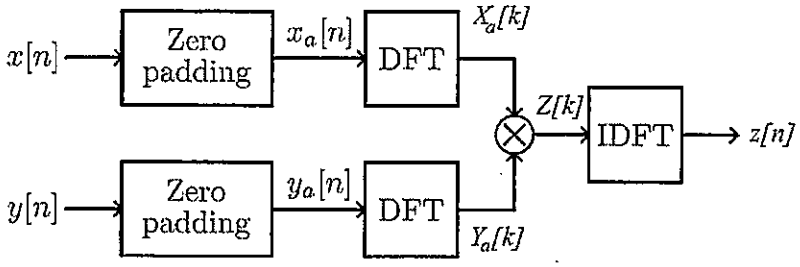


Figure Q4.1

i) Determine the sequences $x_a[n]$ and $y_a[n]$.

[5 marks]

ii) Use the decimation-in-time Fast Fourier Transform (FFT) algorithm as shown in **Figure Q4.2**, compute $X_a[k]$ and $Y_a[k]$.

[10 marks]

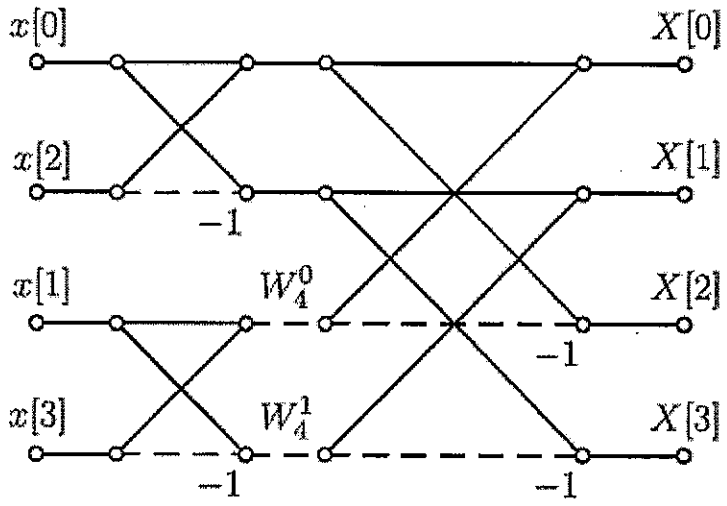


Figure Q4.2

iii) Determine the sequences $Z[k]$ and $z[n]$.

[10 marks]

Continued .....

APPENDIX

SUMMARY OF USEFUL TABLES

TABLE A1. DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Discrete-Time Fourier Transform
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-j\omega n_o}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{r^n \sin\omega_p (n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega} + r^2 e^{-j\omega})}$
$\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases}$	$\frac{\sin[\omega(M+1)/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{n=-\infty}^{\omega} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
$\cos(\omega_0 n + \phi)$	$\sum_{n=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$

Continued.....

TABLE A2. SOME COMMON z-TRANSFORM PAIRS

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

Continued.....

TABLE A3. SOME COMMON WINDOWS FOR FIR FILTER DESIGN

The impulse response of the ideal highpass filter is given by

$$h_i[n] = \delta[n] - \frac{\omega_c}{\pi}\text{sinc}\left(\frac{\omega_c n}{\pi}\right) \qquad n = 0, \pm 1, ..., \pm\infty$$

where ω_c is the 3-dB cutoff frequency.

Rectangular	$w[n] = \begin{cases} 1 & 0 \le n \le N \\ 0 & else \end{cases}$
Hanning	$w[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2n\pi}{N}\right) & 0 \le n \le N \\ 0 & else \end{cases}$
Hamming	$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2n\pi}{N}\right) & 0 \le n \le N \\ 0 & else \end{cases}$
Blackman	$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2n\pi}{N}\right) + 0.08\cos\left(\frac{4n\pi}{N}\right) & 0 \le n \le N \\ 0 & else \end{cases}$
Kaiser	$w[n] = \frac{I_0\left(\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)} \quad 0 \le n \le N \quad \alpha = N/2$ where $\beta = \begin{cases} 0.1102(A-8.7), & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0, & A < 21 \end{cases}$ $N = \begin{cases} (A-7.95)/(2.285\Delta\omega), & A \ge 21 \\ 5.655/\Delta\omega, & A < 21 \end{cases}$

Continued.....

TABLE A4. COMPARISON OF COMMONLY USED WINDOWS

Type of Window (Filter length = N+1)	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Stopband Attenuation, $20\log_{10}\delta$ (dB)	Transition Width ($\Delta\omega$ $\lvert w_p - w_s \rvert$)
Rectangular	-13	$4\pi/(N+1)$	-21	$1.8\pi/N$
Hanning	-31	$8\pi/N$	-44	$6.2\pi/N$
Hamming	-41	$8\pi/N$	-53	$6.6\pi/N$
Blackman	-57	$12\pi/N$	-74	$11\pi/N$

TABLE A5. IIR FILTER DESIGN

TABLE A5.1. THE COEFFICIENTS IN THE SYSTEM FUNCTION OF A NORMALIZED BUTTERWORTH LOW PASS FILTER (Ω_c=1) FOR ORDERS $1 \le N \le 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

TABLE A5.2. CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a| < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

Continued.....

TABLE A6. PARAMETERS OF ANALOG FILTER PROTOTYPES

Discrimination factor $d = \left[\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}\right]^{1/2}$

Selectivity factor $k = \dfrac{\Omega_p}{\Omega_s}$

Butterworth	Order, $N \geq \dfrac{\log d}{\log k}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = \Omega_c e^{j(2k+1+N)\pi/(2N)}$
Chebyshev-I	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = H_a(j0) \prod_{k=0}^{N-1} \dfrac{-s_k}{s - s_k}$ where $s_k = -\Omega_p \sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j\Omega_p \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]$ $\epsilon = \sqrt{(1-\delta_p)^{-2}-1}$ $H_a(j0) = \begin{cases} 1, & \text{odd } N \\ (1+\epsilon^2)^{-0.5}, & \text{even } N \end{cases}$
Chebyshev-II	Order, $N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ System function, $H_a(s) = \prod_{k=0}^{N-1} \dfrac{s_k}{s'_k}\left(\dfrac{s - s'_k}{s - s_k}\right)$ where $s_k = \Omega_s\{-\sinh[\sinh^{-1}(1/\epsilon)/N] \sin[(2k+1)\pi/(2N)]$ $+ j \cosh[\sinh^{-1}(1/\epsilon)/N] \cos[(2k+1)\pi/(2N)]\}^{-1}$ $s'_k = j\Omega_s / \cos((2k+1)\pi/(2N))$ $\epsilon = 1/\sqrt{\delta_s^{-2}-1}$

Continued.....

Elliptic	$N \geq \dfrac{\log(16/d^2)}{\log(1/q)}$, $q = q_o + 2q_o^5 + 15q_o^9 + 150q_o^{13}$, $q_o = \dfrac{1}{2}\left[\dfrac{1-(1-k^2)^{1/4}}{1+(1-k^2)^{1/4}}\right]$

TABLE A7. DISCRETE-TIME FOURIER ANALYSIS

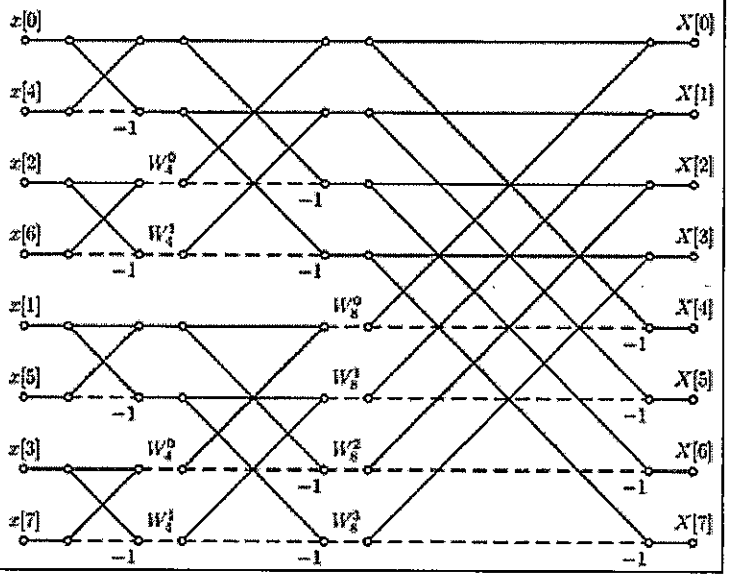
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time FFT



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